actual responses, even when the spin axis drift has violated the small-angle approximation appreciably.


### References

[1] Wheeler, P. C., "Two-pulse attitude control of an asymmetric spinning satellite," AIAA Preprint 63-338 (August 1963).

[2] Suddath, J. H., "A theoretical study of the angular motions of spinning bodies in space," NASA TR R-83 (1960).


# Growth of the Turbulent Inner Wake behind 3-In.-Diam Spheres


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There have been a number of experiments carried out at several laboratories to measure the growth of the turbulent inner wake behind hypervelocity spheres. The available literature indicates that the spheres investigated have not exceeded $\frac{1}{2}$ in. in diameter. For example, Slattery and Clay[1] have studied wakes behind $\frac{1}{4}$- and $\frac{1}{2}$-in.-diam solid aluminum spheres that have been launched at a velocity of 9000 fps into air at pressures ranging from 40 to 760 mm Hg. For the atmosphere data, a $\frac{1}{2}$-power dependence of wake width upon length is apparent. The lower pressure data exhibit $\frac{1}{2}$-power dependence for the first 50 body diameters, and then a $\frac{1}{3}$-power dependence seems to be better. Dana and Short[2] studied the turbulent wake behind 0.22- and 0.31-in.-diam aluminum spheres launched at velocities from 2000 to 9000 fps into atmospheric air. The measured growth rate is in good agreement with the theory of Lees and Hromas.[3] More recently, Murphy and Dickinson[4] have launched $\frac{1}{8}$- and $\frac{1}{4}$-in.-diam spheres at muzzle velocities of 7100 and 6400 fps, respectively, into atmospheric air. From the results, they concluded that 1) the data, by least-squares fit, are well represented by a $\frac{1}{2}$-power dependence of width on length for the entire length of wake; 2) the data can also be represented by a $\frac{1}{3}$-power dependence, which shows a change in the growth rate at about 150–200 body diameters, the initial growth rate being less than the one further downstream; and 3) there may be a scaling effect in the growth rate with projectile size.

In the present experiments, the width of the turbulent inner wake behind 3-in.-diam solid aluminum spheres was measured for wake lengths extending up to 1500 body diameters. The spheres were launched into atmospheric air from a conventional solid propellant gun at a muzzle velocity of 6700 fps. The wake was photographed at three schlieren and ten shadowgraph stations of the CARDE Aeroballistics Range which were triggered simultaneously when the sphere passed a predetermined downrange station.

The data were reduced by measuring the width of the wake from the schlieren and shadowgraph photographs at longitudinal intervals of $\frac{1}{4}$ or $\frac{1}{2}$ body diameter. To even out irregularities in the measured width due to turbulence, the measurements thus obtained were averaged for stretches of wake of approximately 5 body diameters in length and were nondimensionalized with respect to the body diameter for presentation. In order to locate correctly a particular transverse wake dimension relative to the projectile, a correction to the longitudinal wake dimension was applied, as in the experiments of Murphy and Dickinson. The correction accounts for the apparent shortening of the longitudinal dimension due to projectile deceleration. It must also be noted here that, in the analysis of the data, the interaction of the reflected bow shock wave with the turbulent wake, and in particular the effect of this on the growth rate, has not been taken into account. In these experiments, however, the initial 180 body diameters of wake are free of disturbances caused by the reflected bow shock wave.

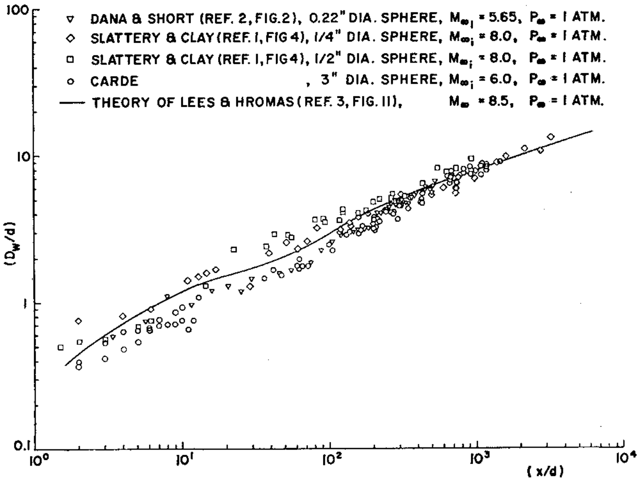


**Fig. 1 The growth of the turbulent inner wake behind spheres (wake width vs wake length).**

Experimental evidence indicates that the growth of the hypersonic turbulent wake can be described by a relationship of the form $(D_w/d) \propto (x/d)^n$. Theoretical analyses indicate that $n = \frac{1}{3}$ is asymptotically approached in the far wake. Figure 1 shows a logarithmic plot of the turbulent wake width $D_w/d$ measured behind various sizes of spheres against the wake length $x/d$. For $x/d < 200$, there is some variation when a set of data from one source is compared with that from another. In the region $x/d < 200$, the 3-in.-diam data seem to follow closely the data of Dana and Short. The data of Slattery and Clay generally fall above in this region. For $x/d > 200$, all of the data agree quite well with each other, and the tendency is to approach a $\frac{1}{3}$-power dependence of width on length. (Unfortunately, the data of Murphy and Dickinson were not available in a form that could be replotted for comparison.)

The same 3-in.-diam sphere data are plotted in Fig. 2, the abscissa in this case being $(x/d)^{1/3}$. In this figure, one of the two presently available theoretical curves calculated by Lees and Hromas for a sphere with $M_\infty = 8.5$, $p_\infty = 1$ atm is also

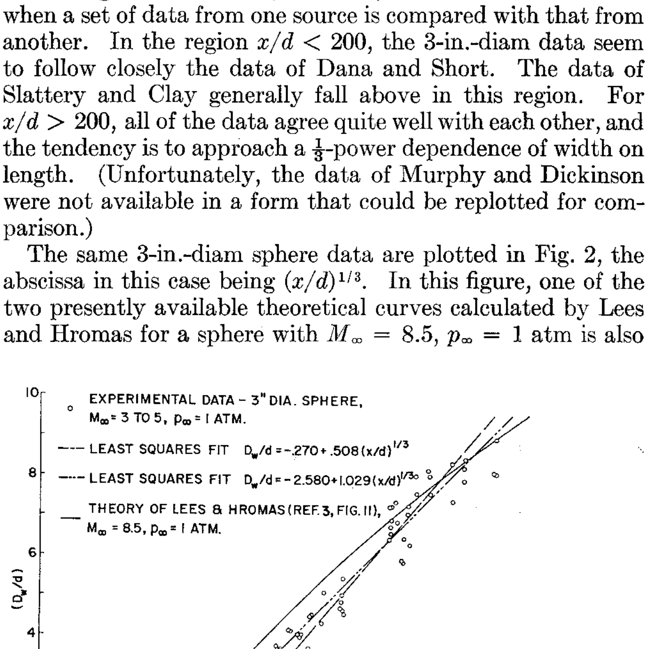


**Fig. 2 The growth of the turbulent inner wake behind spheres (wake width vs wake length$^{1/3}$).**


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presented for comparison. The agreement is good, although the comparison strictly speaking is not appropriate, since the experimental data were obtained at $M_\infty = 3$ to 5, $p_\infty = 1$ atm. Least-squares fit relations are presented for the 3-in. sphere data on Fig. 2 together with those of Murphy and Dickinson for their $\frac{1}{8}$- and $\frac{1}{4}$-in.-diam sphere data. The agreement is reasonable so that there does not appear to be any pronounced size effect, even though the projectile size factor is up to 12.

From the results it can be concluded that, if the wake width data for the 3-in.-diam spheres are averaged over stretches of approximately 5 body diameters in length, the growth rate can be represented by an $(x/d)^{1/2}$ relationship for the entire length of the wake.[5] However, an $(x/d)^{1/3}$ growth relationship can also be used, and it would seem more appropriate to use such a relationship, since the theoretical analyses of the turbulent wake predict this form of growth, whereas none predict the $(x/d)^{1/2}$ relationship. In particular, the analysis of Lees and Hromas for the turbulent wake predicts a growth rate that is in good agreement with the data of this investigation when such data are viewed in the light of an $(x/d)^{1/3}$ growth form. The agreement is good even to the extent of showing the region of the inflection point in the growth curve beyond which there is an increase in the growth rate. This phenomenon arises as a result of the marked increase in the turbulent viscosity when the turbulent front has reached the cooler, denser gas with the high enthalpy gradients in the inviscid flow.

Although the 3-in.-diam spheres represent a factor of up to 12 from the sphere firings at other laboratories, no obvious scaling effect is apparent in the growth rate of the turbulent inner wake, at this pressure and in this Mach number region, when the results are compared.

### References

[1] Slattery, R. E. and Clay, W. G., "Width of the turbulent trail behind a hypervelocity sphere," Phys. Fluids **4**, 1199–1201 (October 1961).

[2] Dana, T. A. and Short, W. W., "Experimental study of hypersonic turbulent wakes," Convair Rept. ZPh-103A (August 1961).

[3] Lees, L. and Hromas, L., "Turbulent diffusion in the wake of a blunt-nosed body at hypersonic speeds," J. Aerospace Sci. **29**, 976–993 (August 1962); also Space Technology Labs., Aerodynamics Dept., R-50 (July 1961).

[4] Murphy, C. H. and Dickinson, E. R., "Growth of the turbulent wake behind a supersonic sphere," AIAA J. **1**, 339–342 (1963).

[5] Knystautas, R., "The growth of the turbulent inner wake behind a 3 inch diameter sphere," Canadian Armament Research and Development Establishment TR 488/64 (February 1964).

# Effect of Shock Curvature on Turbulent Heating of Sphere-Cones

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### Nomenclature

$a$ = speed of sound, fps
$C_D$ = Newtonian nose drag coefficient
$C_m$ = compressibility function [Eq. (7)]
$M$ = Mach number
$Pr$ = Prandtl number
$q$ = convective heat-transfer rate
$R_n$ = nose radius, ft
$r$ = dimensionless body radius
$s$ = dimensionless surface location
$t$ = dimensionless distance from body surface
$u$ = velocity, fps
$x$ = dimensionless axial location
$y$ = dimensionless shock coordinate
$z$ = altitude, ft
$\delta$ = dimensionless boundary-layer thickness
$\theta_c$ = cone half-angle, deg
$\theta_{cs}$ = conical shock angle, deg
$\mu$ = viscosity, lbm/ft-sec
$\rho$ = density, lbm/ft³
$\phi_T$ = function defined by Eq. (6)
$\psi$ = stream function defined by Eq. (2)

***Subscripts***

$c$ = cone
$e$ = boundary-layer edge condition
$ns$ = emanating from a normal shock
$\infty$ = freestream condition
$*$ = reference condition

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A RECENT note[1] presented a simple method for estimating shock curvature effects on the outer edge conditions of a laminar boundary layer. The method was applied to a sphere-cone configuration with the results indicating a significant increase in convective heating over levels computed on the basis of flow emanating through a normal shock. Since, for most re-entering bodies, the majority of the heat pulse occurs with a turbulent boundary layer, an easily applied procedure for determining shock curvature effects in this case is perhaps even more important.

Following the general approach of Ref. 1, a method is developed for determining the shock curvature effects on a zero pressure gradient turbulent boundary layer. Only axisymmetric flows are considered, and the laminar portion of the boundary layer preceding the turbulent is neglected. Then, from continuity considerations,

$$\rho_\infty u_\infty y^2 = 2[\psi(s, \delta)/R_n^2] \tag{1}$$

with

$$\frac{\psi(s, \delta)}{R_n^2} = \int_0^\delta \rho u r dt \tag{2}$$

where all lengths are normalized with respect to the nose radius $R_n$. Following established methods[2] and assuming a $\frac{1}{7}$ power law variation of $\rho u$ through the boundary layer, i.e.,

$$\rho u/\rho_e u_e = (t/\delta)^{1/7} \tag{3}$$

then

$$\psi(s, \delta)/R_n^2 = \tfrac{7}{8}\rho_e u_e r\delta \tag{4}$$

where the boundary-layer thickness can be represented by

$$\delta = \frac{0.382}{\mathrm{Pr}^{2/3}} \frac{R_n^{-1/5}}{\rho_* u_e r} \left[\int_0^s \rho_* u_e \mu_*{}^{1/4} r^{5/4} ds\right]^{4/5} \tag{5}$$

Combining Eqs. (1, 4, and 5) yields the relation

$$r^{5/4}\frac{ds}{dy} = \phi_T(y, \rho_*/\rho_e) = 3.12R_n{}^{1/4}\left(\frac{\rho_\infty a_\infty}{\mu_\infty}\right)^{1/4} \times$$
$$M_\infty{}^{5/4} C_m \frac{y^{3/2}}{(\rho_e/\rho_\infty)(u_e/a_\infty)(\mu_e/\mu_\infty)^{1/4}}\left[1 + \frac{1}{2}\frac{d\ \ln\rho_*/\rho_e}{d\ \ln y}\right] \tag{6}$$

with

$$C_m = [(\rho_*/\rho_e)(\mu_e/\mu_*)]^{1/4} \tag{7}$$

Equation (6) (in direct correspondence with the laminar relation given in Ref. 1) connects the shock-wave coordinate $y$ with the body coordinate $s$ through the boundary-layer edge conditions. A direct solution of the problem is available if three conditions can be met: first, the body surface pressure is constant; second, the quantity $C_m$ is constant; and, finally, the second term in the brackets in Eq. (6) is zero indicating $\phi_T$ is a function of $y$ only. For sphere-cones, the pressure gradient is nearly zero over the range in which the shock